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GENERALIZATION OF THE CLASSICAL FORMULA OF STABILITY OF CYLINDRICAL SHELLS TO THE CASE OF SPIRAL STIFFENERS

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The stability of a cylindrical shell with eccentric spiral stiffeners in the case of axial compression is considered. A simple computational formula for determination of the critical stress has been obtained. The formula contains all the necessary mechanical and geometric characteristics of the shell and the stiffeners and is, in essence, a generalization of the well-known and widely used classical formula for isotropic smooth shells.

Pioneering investigations in the field of stability of cylindrical shells with spiral stiffeners [1, 2] have shown the efficiency of such structures in the sense of minimum weight under the action of a certain class of loads as compared to the regular structures stiffened by frames and stringers. Theoretical results [1] are in good agreement with experimental data [2] for cylindrical shells with a 45-degree spiral stiffener. However, in the above works, use is made of the apparatus of flat-shell theory, which restricts the range of application of the results to moderately long shells.

Solutions of the problems of stability of cylindrical shells with spiral stiffeners under the action of external pressure, which are applicable in calculation of rising shells of any length, have been obtained in [3, 4] based on refined expressions for the parameters of curvature [5] and the load potential. It has been shown that spirally stiffened shells, particularly those of small length, operate much better than the shells stiffened by frames and stringers.

1. In this work, we use the equations of total strain energy to solve the problem of stability of a cylindrical shell with eccentric spiral stiffeners in the case of axial compression. This energy is averaged over the shell and includes the tensile, flexural, and torsional strains with allowance for eccentricity. Differential translational equilibrium equations obtained by the method of variation of the total energy are reduced to a single equation for the normal translation.

The geometry and coordinate axes of a circular cylindrical shell with spiral-type stiffeners making angles of θ and $-\theta$ with the x^* axis are shown in Fig. 1. We have adopted a dimensionless coordinate system: $x = x^*/R$, $y = y^*/R$, $\xi = \xi^*/R$, and $\eta = \eta^*/R$. Variation of the total potential energy of strain of the cylindrical shell with spiral stiffeners by translations u, v, and w yields three equilibrium equations, from which we obtain (by the operator method) the following resolving equation of the problem:

$$\left\{c_{81}\frac{\partial^8}{\partial x^8}+c_{82}\frac{\partial^8}{\partial x^6\partial y^2}+c_{83}\frac{\partial^8}{\partial x^4\partial y^4}+c_{84}\frac{\partial^8}{\partial x^2\partial y^6}+c_{85}\frac{\partial^8}{\partial y^8}+c_{61}\frac{\partial^6}{\partial x^6}+c_{63}\frac{\partial^6}{\partial x^4\partial y^2}+c_{65}\frac{\partial^6}{\partial x^2\partial y^4}+\right.$$

$$+c_{67}\frac{\partial^6}{\partial y^6}+c_{41}\frac{\partial^4}{\partial x^4}+c_{42}\frac{\partial^4}{\partial x^2\partial y^2}+c_{43}\frac{\partial^4}{\partial y^4}+j\frac{\partial^2}{\partial x\partial y}\left(e_{81}\frac{\partial^6}{\partial x^6}+e_{82}\frac{\partial^6}{\partial x^4\partial y^2}+e_{83}\frac{\partial^6}{\partial x^2\partial y^4}+e_{84}\frac{\partial^6}{\partial y^6}+\right.$$

$$\left.+e_{61}\frac{\partial^4}{\partial x^4}+e_{62}\frac{\partial^4}{\partial x^2\partial y^2}+e_{63}\frac{\partial^4}{\partial y^4}+e_{41}\frac{\partial^2}{\partial x^2}+e_{42}\frac{\partial^2}{\partial y^2}\right)+\left[d_{11}\frac{\partial^4}{\partial x^4}+d_{13}\frac{\partial^4}{\partial x^2\partial y^2}+d_{15}\frac{\partial^4}{\partial y^4}+\right.$$

$$\left.\left.+j\frac{\partial^2}{\partial x\partial y}\left(d_{12}\frac{\partial^2}{\partial x^2}+d_{14}\frac{\partial^2}{\partial y^2}\right)\right]N_x\frac{\partial^2}{\partial x^2}\right\}w=0\,. \tag{1}$$

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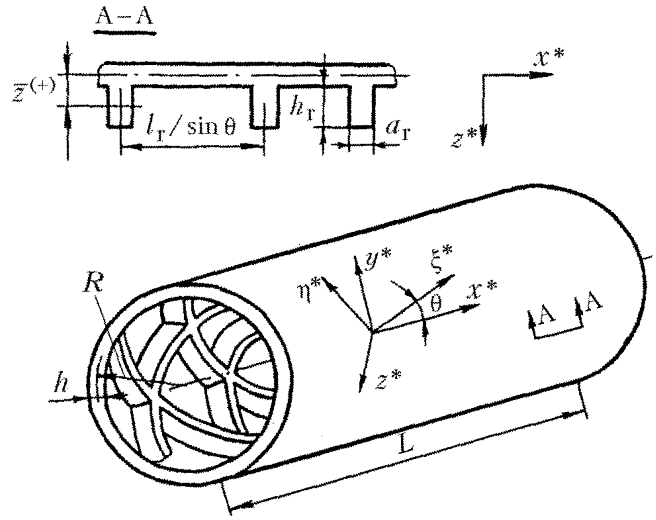

Fig. 1. Geometry and coordinate axes of a cylindrical shell with spiral stiffeners.

We do not give here the coefficients c, e, and d of the resolving equation (1); one may easily write them when [3, 4] is taken into account.

In Eq. (1), we have $j = \sin 2\theta$ for a simple one-dimensional spiral, and the quantities containing the rigidities of the stiffeners in the coefficients c, e, and d must be divided by two. We have $j = \sin 2\theta + \sin(-2\theta) = 0$ for a paired symmetric spiral.

2. For practical calculations it is important to have simple approximate formulas enabling us to evaluate stability in the process of designing. Such a possibility is provided by the use of the semizero-moment theory in the case of shells of moderate and large lengths, to whose equations we pass by applying the strong inequality [6]

$$\frac{\partial^2 f}{\partial y^2} >> \frac{\partial^2 f}{\partial x^2}, \tag{2}$$

where f is any force or strain factor in the shell, to the equations of general theory [5].

After simplifications (following from condition (2)) of the resolving equation (1), we obtain

$$\left\{c_{85}\frac{\partial^8}{\partial y^8} + c_{67}\frac{\partial^6}{\partial y^6} + c_{43}\frac{\partial^4}{\partial y^4} + c_{41}\frac{\partial^4}{\partial x^4} + d_{15}\frac{\partial^2}{\partial y^2}\left[N_x\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2}{\partial y^2} - 1\right)\right]\right\} w = 0\,. \tag{3}$$

Here $c_{67} = 2c_{85}$ and $c_{43} = c_{85}$.

We note that, in Eq. (3), we allow for the influence of circular translations on the value of the potential under the action of compression load for the case of a possible buckling of the shell to form long waves $n = 2$ and $n = 3$ [7].

3. We consider the stability of a stiffened cylindrical shell in axial compression for the case of hinging at its ends.

For paired symmetric spiral elements ($j = 0$), we seek the solution of Eq. (1) in a form corresponding to the asymmetric form of buckling:

$$w = A \sin \lambda_m x \cos ny\,, \quad \lambda_m = m\pi R/L\,. \tag{4}$$

Substitution of (4) into (1) leads to the following expression for determination of the critical load:

$$N_x\lambda_m^2 = \frac{\varphi_1(m, n)}{\varphi_2(m, n)}, \tag{5}$$

where

$$\varphi_1(m,n) = c_{81}\lambda_m^8 + c_{82}\lambda_m^6 n^2 + c_{83}\lambda_m^4 n^4 + c_{84}\lambda_m^2 n^6 + c_{85}n^8 -$$

$$-(c_{61}\lambda_m^6 + c_{63}\lambda_m^4 n^2 + c_{65}\lambda_m^2 + c_{67}n^6) + c_{41}\lambda_m^4 + c_{42}\lambda_m^2 n^2 + c_{43}n^4\,;$$

$$\varphi_2(m,n) = d_{11}\lambda_m^4 + d_{13}\lambda_m^2 n^2 + d_{15}n^4\,.$$

When Eq. (3) of the semizero-moment theory is solved in the form (4), we write an expression for determination of the critical load:

$$N_x = \frac{n^2(n^2-1)^2 c_{85}}{(n^2+1)\,\lambda_m^2 d_{15}} + \frac{\lambda_m^2 c_{41}}{n^2(n^2+1)\,d_{15}}\,. \tag{6}$$

Minimization of expression (6) by λ_m^2 yields the formula for the axial critical load

$$N_x = \frac{2(n^2-1)}{d_{15}(n^2+1)}\sqrt{c_{85}c_{41}}\,.$$

Hence, disregarding small terms of the order h/R as compared to unity, for shells with paired symmetric stiffeners ($j = 0$) we obtain the computational formula for determination of the critical stress

$$\sigma_{\mathrm{cr}} = \frac{1}{\sqrt{3(1-\nu^2)}}\,E\,\frac{h}{R}\sqrt{k_1k_2}\,\frac{n^2-1}{n^2+1}\,, \tag{7}$$

where

$$k_1 = \frac{1 + \dfrac{2A_{\mathrm{r}}}{hl_{\mathrm{r}}}\left(1 - \dfrac{1+\nu}{2}\sin^2 2\theta\right)}{1 + \dfrac{2(1-\nu^2)A_{\mathrm{r}}\sin^4\theta}{hl_{\mathrm{r}}}}\,;\quad k_2 = 1 + \frac{12(1-\nu^2)}{h^3}\left[\frac{J_{\mathrm{r}}\sin^2 2\theta}{4(1+\nu)\,l_{\mathrm{r}}} + \frac{2J_{\mathrm{sk.r}}\sin^4\theta}{l_{\mathrm{r}}}\right].$$

Here $J_{\mathrm{sk.r}} = J_\eta + \dfrac{\bar{z}^2 A_{\mathrm{r}}}{1 + \dfrac{2(1-\nu^2)A_{\mathrm{r}}\sin^4\theta}{hl_{\mathrm{r}}}}$ is the moment of inertia of the stiffening member of the rib, which has been computed with allowance for the joint work with the skin belt, $J_{\mathrm{r}} = A_{\mathrm{r}}^4(4\pi^2 J_{\mathrm{p}})^{-1}$ is the Saint Venant torsion constant for the stiffening member, and $J_{\mathrm{p}} = J_\eta + J_z$ is the polar moment of inertia of the spiral rib.

For a smooth shell, the classical formulas [7]

$$\sigma_{\mathrm{cr}} = \frac{1}{\sqrt{3(1-\nu^2)}}\,\frac{n^2-1}{n^2+1}\,E\,\frac{h}{R} \approx 0.605\,\frac{n^2-1}{n^2+1}\,E\,\frac{h}{R}\,,\quad \sigma_{\mathrm{cr}} \approx 0.605\,E\,\frac{h}{R}\quad \text{for}\quad n \gg 1 \tag{8}$$

follow as a particular case of formula (7).

4. We consider, as an example, a cylindrical shell with radius R = 100 mm and thickness h = 1 mm, having external and internal stiffeners. The stiffening members have been taken in the form of ribs with a rectangular cross section of height $h_{\mathrm{r}} = 4h$ = 4 mm and width $a_{\mathrm{r}} = 2h$ = 2 mm. The distance between the ribs is l_{r} = 10–30 mm. The results of a comparison of the critical loads for shells with spiral and regular stiffeners on condition that the stiffening

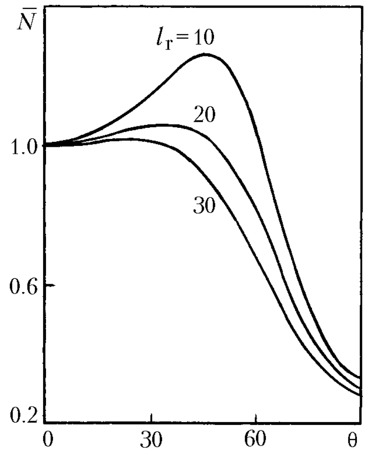

Fig. 2. Influence of the parameters of a spirally stiffened cylindrical shell on the value of the axial critical load.

members of the stiffeners compared have the same mass are presented in Fig. 2. Here the ratio of the critical loads $\bar{N} = N_{cr}^{spir}/N_{cr}^{str}$ for shells with spiral (N_{cr}^{spir}) and stringer (N_{cr}^{str}) stiffeners of regular type is plotted on the ordinate axis, and the slope θ is plotted on the abscissa axis. The optimum slope of the spiral members was calculated for different values of θ from 0 to 90^o. It was assumed that the length of the shell is $L/R = 5$; however, these results also hold true for shells of another length if we take into account the results [7] referring to the range of applicability of the classical formulas (8).

In the case of the action of axial compression, the advantage of spiral stiffeners over regular ones is usually preserved for shells of any length.

NOTATION

A_r, cross-sectional area of the rib, mm^2; a_r, width of the rib, mm; E, elastic modulus of the shell material, daN/mm^2; h_r, height of the rib, mm; J_η and J_z, moments of inertia of the cross section of the spiral rib, mm^4; L, length of the shells, mm; l_r, distance between two neighboring parallel spirals along the normal, mm; m, No. of the harmonic in the longitudinal direction of the shell; N, axial load, daN/mm; n, number of total waves in the circular direction; R, radius of the shell, mm; x, y, z, coordinate system of the shell (longitudinal, circular, and normal coordinate axes); ξ, η, coordinate system of the cross section of the rib; θ, slope of spiral stiffeners, deg; ν, Poisson coefficient of the shell material; σ_{cr}, axial critical stress, daN/mm^2. Subscripts and superscripts: cr, critical; spir, spiral; str, stringer; r, rib; p, polar; sk.r, ribs with the skin belt relative to the median of the skin.

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